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A NEW FORMULATION OF THE BOUNDARY INTEGRAL EQUATIONS OF THE FIRST KIND IN ELECTROELASTICITY†

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A system of boundary integral equations of the first kind with piecewise-smooth kernels, to which the boundary-value problems of electroelasticity reduce in the case of steady-state oscillations, is formulated. The proposed approach does not use the idea of fundamental solutions and is based solely on an analysis of the characteristic polynomial of the electroelasticity operator.

The extension of the method of boundary integral equations to models describing the connected fields in continuum mechanics—electroelasticity [1, 2], magnetoelectroelasticity [3], and thermoelectroelasticity [4], is based, as a rule, on potential theory and the reciprocity theorem, on the construction on fundamental and singular solutions for the corresponding operators, and generalized limit theorems for analogues of the potentials of a single and a double layer [5]. Whereas in the isotropic theory of elasticity these solutions are expressed in explicit form in terms of elementary or special functions, for the models of connected problems indicated, only integral representations of the fundamental solutions can be constructed, which, to a considerable extent, reduces the effectiveness of any further numerical analysis of the systems of boundary integral equations constructed based on the boundary-element method [6].

Another approach to the formulation of systems of boundary integral equations of the anisotropic theory of elasticity was proposed in [7], which enables the boundary-value problem for a finite body to be reduced to a system of boundary integral equations of the first kind with piecewise-smooth kernels without using fundamental solutions. This approach is based solely on the well-known properties of the analyticity of the Fourier transforms of functions with a carrier in a limited region, and an analysis of the characteristic polynomial of the corresponding operator and leads to a system of boundary integral equations in the unit circle (the plane case) or the unit sphere (the three-dimensional case). A formulation of the corresponding boundary integral equations for problems of the isotropic theory of elasticity and acoustics was given in [8, 9], and also a numerical construction of the corresponding inverse operators by a combination of the boundary-element method and the Tikhonov regularization method. An important advantage of the proposed algorithm is the fact that the coefficients of the algebraic systems obtained can be calculated in explicit form, rather than in the form of single or double integrals, as in the classical version of boundary equations. In view of the fact that the procedure of inverting a Fredholm operator equation of the first kind is ill-posed, this scheme requires regularization [10] in some form.

Note that this approach cannot be applied directly to operators, characteristic of polynomials which contains a zero component (the Laplace operator, the static theory of elasticity and electroelasticity).

Below we propose an extension of the method of boundary integral equations of the first kind to problem of electroelasticity. The problems involved in the numerical realization are discussed, and numerical examples are given. Note that the proposed system of boundary integral equations enables the oscillations of an electroelastic medium to be analysed when there is attenuation (within the framework of the theory of complex moduli), and this approach is particularly effective when it is only required to investigate boundary characteristics (the displacements of boundary points, the potential difference, the charge distribution under an electrode, etc.).

1. FORMULATION OF THE PROBLEM AND THE SETTING UP OF THE SYSTEM OF BOUNDARY INTEGRAL EQUATIONS

Suppose a bounded simply connected region $V \in R^n$ ($n = 2, 3$), stellar with respect to a certain sphere, is bounded by a piecewise-smooth surface S. The region V is occupied by an electroelastic medium,

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which performs steady oscillations with frequency ω due to the action of a potential difference $2\phi_0$, applied to a pair of electrodes S_+ and S_-. We will assume that the remaining part of the boundary S_D is not an electrode; the whole boundary S is stress-free. Note that the general approach to obtaining boundary integral equations does not change if we consider the problem of the action on the body V of mechanical loads, including contact-type loads (and also in the case of the connection of a piezoelectric element in a certain electric circuit), and also the problem of kinematic or mixed mechanical boundary conditions.

The boundary-value problem is described by the following system of equations [11]

$$\begin{gathered}\sigma_{mj,j} + \rho\omega^2 u_m = 0, \quad D_{m,m} = 0 \\ \sigma_{mj} = c_{mjkl}u_{k,l} + e_{kmj}\phi_{,k}, \quad D_m = e_{mkl}u_{k,l} - \ni_{mk}\phi_{,k}\end{gathered} \tag{1.1}$$

with boundary conditions

$$\sigma_{ij}n_j|_S = 0, \quad \phi|_{S_\pm} = \pm\phi_0, \quad D_m n_m|_{S_D} = 0 \tag{1.2}$$

Here u_m, D_m and n_j are the components of the displacement vectors, the electric induction and the unit outward normal, respectively, σ_{mj}, c_{mjkl}, e_{kmj} and $\ni_{mk}$ are the components of the stress tensors, the elasticity constants, the piezoelectric constants, and the permittivities, ϕ is the electric potential and ρ is the density.

We will apply a Fourier integral transformation to Eqs (1.1); eliminating the transforms of the stresses and the induction, we obtain the system

$$\begin{gathered}(c_{mjkl}\alpha_j\alpha_l - \rho\omega^2\delta_{mk})U_k(\alpha) + e_{kmj}\alpha_j\alpha_k\Phi(\alpha) = V_m(\alpha), \\ e_{mkl}\alpha_m\alpha_l U_k(\alpha) - \ni_{mj}\alpha_m\alpha_j\Phi(\alpha) = V_4(\alpha)\end{gathered} \tag{1.3}$$

where

$$\begin{gathered}V_m(\alpha) = \int_S [\sigma_{mj}n_j - i\alpha_j(c_{mjkl}u_k + e_{lmj}\phi)n_l]e^{i(\alpha,x)}dS \\ U_k(\alpha) = \int_V u_k(x)e^{i(\alpha,x)}dV, \quad \Phi(\alpha) = \int_V \phi(x)e^{i(\alpha,x)}dV \\ V_4(\alpha) = \int_S [D_m n_m - i\alpha_m(e_{mkl}u_k - \ni_{ml}\phi)n_l]e^{i(\alpha,x)}dS\end{gathered} \tag{1.4}$$

Solving system (1.3) for the components $U_k(\alpha)$ and $\Phi(\alpha)$, we obtain

$$U_k(\alpha) = \frac{p_{km}(\alpha,\omega)V_m(\alpha)}{p_0(\alpha,\omega)}, \quad \Phi(\alpha) = \frac{p_{4m}(\alpha,\omega)V_m(\alpha)}{p_0(\alpha,\omega)}, \quad m = 1,2,3,4 \tag{1.5}$$

$$p_0(\alpha, \omega) = \det(A(\alpha, \omega))$$

where $A = \|A_{mk}, (\alpha, \omega)\|$ is the matrix of linear system (1.3) with respect to unknowns $U_k(\alpha)$ and $\Phi(\alpha)$: its elements can be represented in the form

$$\begin{gathered}A_{mk}(\alpha, \omega) = a_{mjkl}\alpha_j\alpha_l - \omega^2 b_m\delta_{mk} \\ b_1 = b_2 = b_3 = \rho, b_4 = 0, a_{mjkl} = c_{mjkl}, \quad m,k = 1,2,3 \\ a_{mj4l} = a_{4jml} = e_{lmj}, \quad m = 1,2,3, \quad a_{4j4l} \doteq -\ni_{jl}\end{gathered}$$

$p_{km}(\alpha, \omega)$ are the cofactors of the elements of the matrix $A(\alpha, \omega)$, which are sixth-order polynomials in the components α_j, and $p_0(\alpha, \omega)$ is an eighth-order polynomial in α_j.

The polynomial $p_0(\alpha, \omega)$ will be called the characteristic polynomial of the electroelasticity operator. This polynomial has eight complex manifolds of zeros (in the plane case when $n = 2$ there are six) $\alpha_{3j}^{\pm} = \pm\alpha_{3j}(\alpha', \omega)$ ($j = 1, 2, 3, 4$), $\alpha' = (\alpha_1, \alpha_2)$, while for the electroelastic medium among these manifolds there is the origin of coordinates, and the previously used approach [7] cannot be applied directly in this case (one of the boundary equations becomes the condition for the boundary-value problem

$$\int_S D_n dS = 0$$

to be solvable and does not enable the system of boundary integral equations to be closed).

However, by analysing relations (1.5) we find that there right-hand sides are irregular when $\alpha_3 = \pm\alpha_{3j}(\alpha', \omega)$, while the left-hand sides are analytic functions for $u_j(x)$, $\phi(x) \in W^1{}_2(V)$. Hence, to eliminate the contradiction which arises we need to require that the following equalities should be satisfied

$$p_{km}(\alpha', \pm\alpha_{3j}(\alpha', \omega), \omega)V_m(\alpha', \pm\alpha_{3j}(\alpha', \omega)) = 0 \quad (1.6)$$
$$k, j = 1, 2, 3, 4, \ \alpha' \in \mathbb{C}^{n-1}$$

It is easy to see that part of relations (1.6) is a consequence of the remaining ones, and we will therefore henceforth assume that $k = 1$. These relations are unique conditions of solvability of the initial boundary-value problem, connecting the boundary values of the unknowns $u_i|_S$, $D_n\ |S_\pm$, $\phi|_{SD}$ and the potential ϕ_0

$$\int_{S_+} G_{1j}^{\pm}(\alpha', x)D_+(x)dS_x + \int_{S_-} G_{1j}^{\pm}(\alpha', x)D_-(x)dS_x + \int_S G_{2mj}^{\pm}(\alpha', x)u_m(x)dS_x +$$
$$+ \int_{S_D} G_{24j}^{\pm}(\alpha', x)\phi(x)dS_x = \phi_0 H_j^{\pm}(\alpha') \quad (1.7)$$

where

$$G_{1j}^{\pm}(\alpha', x) = p_{14}(\alpha, \omega)e^{i(\alpha, x)}, \quad G_{2mj}^{\pm}(\alpha', x) = -i\alpha_s n_l a_{ksml} p_{1k}(\alpha, \omega)e^{i(\alpha, x)}$$
$$\text{for } \alpha_3 = \pm\alpha_{3j}(\alpha', \omega), \ j = 1, 2, 3, 4, \ m = 1, 2, 3, 4 \quad (1.8)$$
$$H_j^{\pm}(\alpha') = -\int_{S_+} G_{24j}^{\pm}(\alpha', x)dS_x + \int_{S_-} G_{24j}^{\pm}(\alpha', x)dS_x$$

Here $p_{1m}(\alpha, \omega)$ are the cofactors of the elements of the first row of the matrix $A_{mk}(\alpha, \omega)$.

System (1.7) is a system of boundary integral equations of the first kind with piecewise-smooth kernels, and discontinuities can only occur along the lines of change of the boundary conditions and on the irregular lines on the boundary S. Hence, (1.7) generate a completely continuous operator, which converts the function from a certain set $Q(S)$ into smooth functions of $\alpha' \in \mathbb{C}^{n-1}$. In the general case, the procedure for inverting such an operator is ill-posed in view of the fact that the operator, inverse to the completely continuous operator [10] is unbounded. In this case, in view of the special form of the right-side of (1.7), the problem of such inversion is conditionally well-posed and allows of an effective procedure of numerical inversion, which is based on a combination of the main ideas of the boundary-element method and the Tikhonov regularization method [10]. At the first stage the boundary S is approximated by a polyhedron, the faces of which we will henceforth call the elements. Within each element the unknown functions are interpolated in terms of the nodal unknowns, and an algebraic system is then set up on the basis of boundary equations (1.7) using the collocation method. Systems of this type are ill-conditioned in view of the complete continuity of the integral operator on the left-hand side of (1.7) and require regularization when inverted.

We will illustrate this technique in more detail using two simple examples of the cases most often encountered in practice of analysing electroelastic bodies for a class 6 mm piezoelectric ceramics polarized along the x_3 axis.

2. THE BOUNDARY INTEGRAL EQUATIONS FOR ANTIPLANE DEFORMATION

Suppose S is the interior of a cylindrical region with boundary $S = L \times R_1$, where L is the boundary of the bounded simply connected region in the x_1x_2 plane, where $L = L_\pm \cup L_D$, and $L_\pm$ is the electroded part of the boundary. We will assume that $u_1 = u_2 = 0$, $u_3 = u(x_1, x_2)$, $\phi = \phi(x_1, x_2)$. The system of equations of electroelasticity has the form [11]

$$c_{44}\Delta u + e_{15}\Delta\phi + \rho\omega^2 u = 0, \quad e_{15}\Delta u - \text{э}_{11}\Delta\phi = 0 \quad (2.1)$$

We will further consider the case when the boundary L is stress-free, and oscillations are excited by the potential difference on the electrodes $L_\pm$, which corresponds to the following boundary conditions

$$c_{44}\frac{\partial u}{\partial n}+e_{15}\frac{\partial \phi}{\partial n}\bigg|_{L}=0,\quad e_{15}\frac{\partial u}{\partial n}-э_{11}\frac{\partial \phi}{\partial n}\bigg|_{L_D}=0$$
$$\phi\,|_{L_\pm}=\phi_0 \tag{2.2}$$

In this case

$$p_0(\alpha,\omega)=(\alpha_1^2+\alpha_2^2)[э_{11}\rho\omega^2-(э_{11}c_{44}+e_{15}^2)(\alpha_1^2+\alpha_2^2)]$$

is a fourth-degree polynomial and the corresponding manifolds can be found explicitly

$$\alpha_{21}^{\pm}=\pm i\alpha_1,\quad \alpha_{22}^{\pm}=\pm i\sqrt{\alpha_1^2-k_*^2};\ k_*^2=\frac{э_{11}\rho\omega^2}{э_{11}c_{44}+e_{15}^2} \tag{2.3}$$

and the boundary equations of the form (1.7) with respect to the unknowns $u\,|_L$, $\partial u/\partial n\,|_{L_\pm}$ have the following structure

$$\int_{L_\pm}\left[G_{1j}^{\pm}(\alpha_1,x)\frac{\partial u}{\partial n}+G_{2j}^{\pm}(\alpha_1 x)\frac{\partial \phi}{\partial n}\right]dL_x+\int_L G_{23j}^{\pm}(\alpha_1,x)u(x)dL_x+$$
$$+\int_{L_D} G_{24j}^{\pm}(\alpha_1,x)\phi(x)dL_x=\phi_0 H_j^{\pm}(\alpha_1) \tag{2.4}$$
$$j=1,2;\ \alpha_1\in\mathbb{C}^1$$

In this case the kernels of the characteristic operators can be represented in the form

$$G_{11}^{\pm}(\alpha_1,x)=e_{15}\exp[i\alpha_1(x_1\pm ix_2)],\quad G_{21}^{\pm}(\alpha_1,x)=-э_{11}\exp[i\alpha_1(x_1\pm ix_2)]$$
$$G_{231}^{\pm}(\alpha_1,x)=-\frac{\partial}{\partial n}G_{11}^{\pm}(\alpha_1,x),\quad G_{241}^{\pm}(\alpha_1,x)=-\frac{\partial}{\partial n}G_{21}^{\pm}(\alpha_1,x)$$
$$H_1^{\pm}(\alpha_1)=\left(\int_{L_-}-\int_{L_+}\right)G_{241}^{\pm}(\alpha_1,x)dL_x \tag{2.5}$$
$$G_{12}^{\pm}(\alpha_1,x)=\exp[i(\alpha_1x_1\pm i\sqrt{\alpha_1^2-k_*^2}x_2)],\quad G_{232}^{\pm}(\alpha_1,x)=-\frac{\partial}{\partial n}G_{12}^{\pm}(\alpha_1,x)$$
$$G_{22}^{\pm}(\alpha_1,x)=0,\quad G_{242}^{\pm}(\alpha_1,x)=0,\quad H_2^{\pm}(\alpha_1)=0$$

The solution is constructed by a combination of the boundary-elements method and the regularization method. We will assume that the boundary of the region L is divided into N elements. At the first stage the boundaries $L_\pm$ and L_D are approximated by the dashed lines

$$L_\pm=\bigcup_{q=1}^{N_1}L_q,\quad L_D=\bigcup_{q=N_1+1}^{N_2}L_q$$

where L_q has the following parameterization $x=x_q^0+\beta_q t$, and its parameters area expressed in terms of the coordinates of the ends of qth element using the formulae

$$x_{qj}^0=(x_{qj}+x_{q+1j})/2,\quad \beta_{qj}=(x_{q+1j}-x_{qj})/2,\quad j=1,2$$

Further, in the simplest version, on each of the elements L_q we assume

$$u\,|_{L_q}=u_q,\quad \partial u/\partial n\,|_{L_q}=v_q,\quad \partial\phi/\partial n\,|_{L_q}=\psi_q,\quad \phi\,|_{L_q}=\phi_q$$

and these nodal unknowns satisfy the following system of linear algebraic equations

$$\sum_{q=1}^{N_1+N_2}B_{1pqj}^{\pm}u_q+\sum_{q=1}^{N_1}(A_{1pqj}^{\pm}v_q+A_{2pqj}^{\pm}\psi_q)+\sum_{q=N_1+1}^{N_1+N_2}B_{2pqj}^{\pm}\phi_q=\phi_0H_j^{\pm}(\alpha_{1p})$$
$$p=1,2,\ldots,\quad N_1+N_2,\ j=1,2 \tag{2.6}$$

where the coefficients of the system are found explicitly in terms of the functions $I_{q1}^{\pm}(\alpha_1)$, $I_{q2}^{\pm}(\alpha_1)$

$$I_{q1}^{\pm}(\alpha_1) = I_q(\alpha_1, \pm i\alpha_1), \quad I_{q2}^{\pm}(\alpha_1) = I_q(\alpha_1, \pm i\sqrt{\alpha_1^2 - k_*^2})$$
$$I_q(\alpha_1,\alpha_2) = \int_{L_q} \exp[i(\alpha,x)]dL_x = \frac{\exp[i(\alpha,x_{q+1})] - \exp[i(\alpha,x_q)]}{i(\alpha,\beta_q)} \tag{2.7}$$

For example

$$A_{1pqj}^{\pm} = e_{15} I_{qj}^{\pm}(\alpha_{1p}), \quad j = 1,2; \quad B_{1pq2}^{\pm} = (\alpha_2^{\pm}, n_q) I_{q2}^{\pm}(\alpha_1)$$
$$\alpha_{1p} \in \mathbb{C}^1, \quad \alpha_2^{\pm} = (\alpha_1, \pm i\sqrt{\alpha_1^2 - k_*^2}) \quad n_q = (\beta_{2q}, -\beta_{1q})$$

Thus, system (2.6) is an ill-conditioned system, but its right-hand sides are operators of the same form as the operators on the left-hand side. The simplest Tikhonov regularization of system (2.6) enables us to construct a fairly stable solution.

We will carry out a series of calculations for the rectangle $[0, a] \times [0, b]$, the sides of which $L_{\pm}$ are electroded and to which a potential difference $2\phi_0$ is applied, where

$$L_+ = \{0 \leqslant x_1 \leqslant a, \; x_2 = b\}, \quad L_- = \{0 \leqslant x_1 \leqslant a, \; x_2 = 0\}$$

Here the regularization parameter α has been varied from 10^{-4} to 10^{-7}, the wave number k_* has been varied from 0 to 12, the number of elements N has been varied from 16 to 32, and the collocation points α_{1p}, which belong to the real axis, have been varied, and also a comparison has been carried out with the exact solution

$$u(x_1,x_2) = \frac{\phi_0 \exists_{11}}{\Delta e_{15}} \delta \sin(k_* x_2 - b_1)$$
$$\phi(x_1,x_2) = \frac{\phi_0}{\Delta}[\delta \sin(k_* x_2 - b_1) - (1+\delta)(k_* x_2 - b_1)\cos(b_1)] \tag{2.8}$$
$$\Delta = \delta \sin(b_1) - b_1(1+\delta)\cos(b_1), \quad b_1 = \frac{k_* b}{2}, \quad \delta = \frac{e_{15}^2}{\exists_{11} c_{44}}$$

In Fig. 1(a) we show the distribution of the boundary values of the displacement u_3 for *TsTS*–19 ceramics [11] for $k_* a = 4$ and $a = b = 1$: curves 1 and 2 correspond to $u_3(x_1, 0)$ and $u_3(x_1, b)$, and curve 3 corresponds to

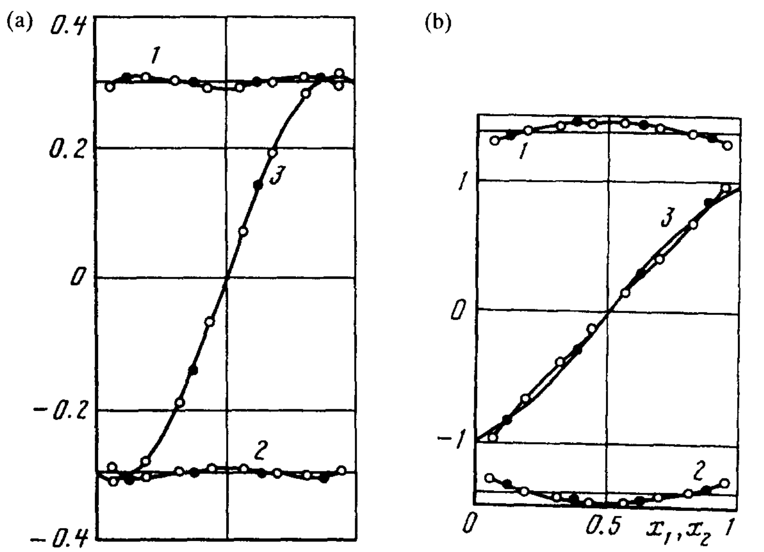

Fig. 1.

$u_3(0, x_2)$. In Fig. 1(b) we show similar distributions for the boundary values of the induction $D_2(x_1, 0)$ and $D_2(x_1, b)$ (curves 1 and 2, respectively) and the potential $\Phi(0, x_2)$ (curve 3). The continuous curves represent the exact solution (2.8), the open points denote the approximate solution for $N = 16$, and the solid points are for $N = 32$. Note the fairly good agreement between the exact and the approximate solutions; the relative error does not exceed 5%.

3. THE BOUNDARY INTEGRAL EQUATIONS FOR PLANE DEFORMATION

Consider the plane deformation of an electroelastic medium, polarized along the Ox_3 axis. Suppose V is a cylindrical region with generatrix parallel to the Ox_2 axis and directrix L. We will assume that $u_1 = u_1(x_1, x_3)$, $u_2 = 0$, $u_3 = u_3(x_1, x_3)$, $\phi = \phi(x_1, x_3)$.

In this case $p_0(\alpha, \omega) = \det A$ is a bicubic polynomial in α_j, and the elements of the matrix A have the form

$$\begin{aligned} &A_{11} = c_{11}\alpha_1^2 + c_{44}\alpha_3^2 - \rho\omega^2, \quad A_{12} = A_{21} = (c_{44} + c_{13})\alpha_1\alpha_3 \\ &A_{13} = A_{31} = (e_{15} + e_{31})\alpha_1\alpha_3, \quad A_{22} = c_{44}\alpha_1^2 + c_{33}\alpha_3^2 - \rho\omega^2 \\ &A_{23} = A_{32} = e_{15}\alpha_1^2 + e_{33}\alpha_3^2, \quad A_{33} = -\text{э}_{11}\alpha_1^2 - \text{э}_{33}\alpha_3^2 \end{aligned} \tag{3.1}$$

In Fig. 2 we show the real and imaginary parts of the manifold $\alpha_{3j}^{+}(\alpha_1, \sqrt{(c_{33}/\rho)})$ $(j = 1, 2, 3)$ for *TsTS*–19 piezoceramics (α_1 runs through sections of the real axis).

Relations (1.4) in this case have the form

$$V_1 = \int_L (\sigma_{11}n_1 + \sigma_{13}n_3 - i((\alpha_1 n_1 c_{11} + \alpha_3 n_3 c_{44})u_1 + (\alpha_1 n_3 c_{13} + \alpha_3 n_1 c_{44})u_3 +$$
$$+(\alpha_1 n_3 e_{31} + \alpha_3 n_1 e_{15})\phi)e^{i(\alpha,x)}dL_x$$
$$V_2 = \int_L (\sigma_{31}n_1 + \sigma_{33}n_3 - i((\alpha_1 n_3 c_{44} + \alpha_3 n_1 c_{13})u_1 + (\alpha_1 n_1 c_{44} + \alpha_3 n_3 c_{33})u_3 +$$
$$+(\alpha_1 n_1 e_{15} + \alpha_3 n_3 e_{33})\phi)e^{i(\alpha,x)}dL_x$$
$$V_3 = \int_L (D_1 n_1 + D_3 n_3 - i((\alpha_1 n_3 e_{15} + \alpha_3 n_1 e_{31})u_1 + (\alpha_1 n_1 c_{15} + \alpha_3 n_3 e_{33})u_3 -$$
$$-(\alpha_1 n_1 \text{э}_{11} + \alpha_3 n_3 \text{э}_{33})\phi)e^{i(\alpha,x)}dL_x$$

The system of integral equations (1.7) retains its form with the replacement $S \to L$, $S_\pm$, $S_D \to L_D$; the kernels of the integral operators have the form

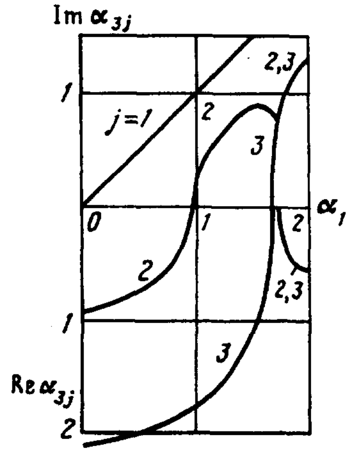

Fig. 2.

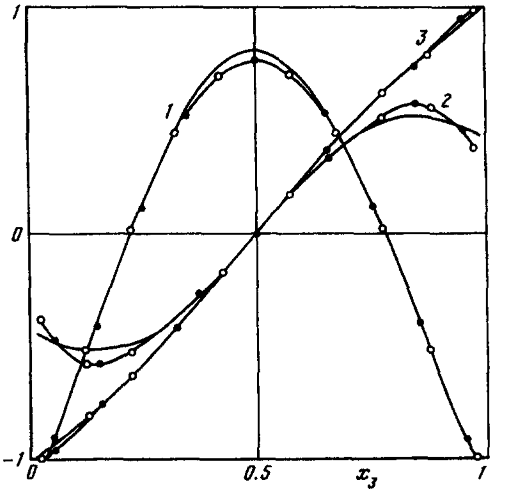

Fig. 3.

$$G_{1j}^{\pm}(\alpha',x)=G_{2j}^{\pm}(\alpha',x)=p_{13}(\alpha_1,\alpha_{3j}^{\pm}(\alpha_1,\omega),\omega)R^{\pm}(\alpha_1,\omega,x_1,x_3)$$

$$G_{2kj}^{\pm}(\alpha',x)=-i\sum_{m=1}^{3}p_{1m}(\alpha_1,\alpha_{3j}^{\pm}(\alpha_1,\omega),\omega)q_{km}(\alpha_1,\alpha_{3j}^{\pm},n_1,n_3)R^{\pm}(\alpha_1,\omega,x_1,x_3),\quad k=1,3,4 \tag{3.2}$$

$$G_{22j}^{\pm}(\alpha',x)=0,\quad \alpha_1\in\mathbb{C}^1$$

where

$$p_{11}(\alpha,\omega)=A_{22}A_{33}-A_{23}^2,\quad p_{12}(\alpha,\omega)=A_{21}A_{33}-A_{13}A_{32}$$

$$p_{13}(\alpha,\omega)=A_{21}A_{32}-A_{31}A_{22},\quad R^{\pm}(\alpha_1,\omega,x_1x_3)=\exp[i(\alpha_1x_1+\alpha_{3j}^{\pm}(\alpha_1,\omega)x_3)$$

$$q_{11}=c_{11}\alpha_1n_1+c_{44}\alpha_3n_3,\quad q_{12}=c_{44}\alpha_1n_3+c_{13}\alpha_3n_1,\quad q_{13}=e_{15}\alpha_1n_3+e_{31}\alpha_3n_1$$

$$q_{31}=c_{13}\alpha_1n_3+c_{44}\alpha_3n_1,\quad q_{32}=c_{44}\alpha_1n_1+c_{33}\alpha_3n_3,\quad q_{33}=e_{15}\alpha_1n_1+e_{33}\alpha_3n_3$$

$$q_{41}=e_{31}\alpha_1n_3+e_{15}\alpha_3n_1,\quad q_{42}=e_{15}\alpha_1n_1+e_{33}\alpha_3n_3,\quad q_{43}=-(\text{э}_{11}\alpha_1n_1+\text{э}_{33}\alpha_3n_3)$$

Discretization of this system of boundary integral equations is carried out in the same way as described above for the antiplane problem. The nodal unknowns in this case are the quantities u_{mq}, ϕ_q, D_{nq}, and the coefficients of the linear algebraic system are found in the same way as (2.7) in the form of explicit formulae.

We will consider two mixed problems as numerical examples which illustrate the use of the proposed approach. The mixed problem of the oscillations of a rectangle $L=[0,a]\times[0,b]$ with boundary conditions

$$x_1=0,\ a{:}\ u_1=0,\ \sigma_{13}=0,\ D_1=0$$

$$x_3=0,\ b{:}\ \sigma_{33}=\sigma_{13}=0,\ \phi=\mp\phi_0$$

This problem has an exact solution, which is represented in Fig. 3 by the continuous curve for $ka=\omega a\sqrt{(\rho/c_{33})}=5$, $a=b=1$; curves 1, 2 and 3 correspond to a $\sigma_{11}(a,x_3)$, $u_3(a,x_3)$, $\phi(a,x_3)$, where the light points denote the numerical solution for $N=40$ and the dark points are for $N=80$.

In addition to the boundary values of the unknowns, this approach also enables one to determine the resonance frequencies. Calculations show that even for a small number of boundary elements, the first resonance frequencies are determined quite accurately (the error in determining the first three resonance frequencies is less than 1%).

The mixed problem of the oscillations of a rectangular trapezium with boundary conditions

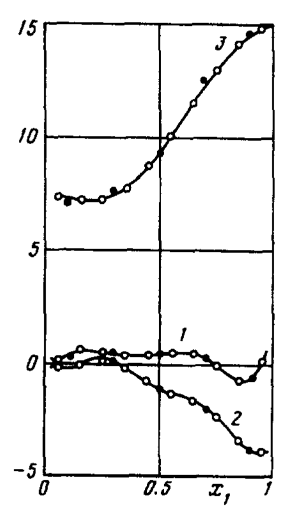

Fig. 4.

$$x_1 = 0: \ u_1 = u_3 = 0, \ D_1 = 0$$

$$x_3 = 2 - 2x_1, \ \tfrac{1}{2} < x_1 < 1: \ \sigma_{ij} n_j = 0 \ (i = 1, 3), \ D_n = 0$$

$$x_3 = 0, b: \ \sigma_{33} = \sigma_{13} = 0, \ \phi = \mp\phi_0$$

In Fig. 4 for $kb = 2$ and $b = 1$ we show values of the horizontal and vertical displacements and the normal component of the electric induction on the lower base of the trapezium for $N = 80$ (curves 1, 2 and 3 respectively), where the light points denote the results of calculations for $N = 20$, and the dark points are for $N = 40$. The results of the calculations confirm that the boundary values of the physical fields have been found quite stably and confirm the internal convergence of the method when the accuracy of the approximation of the integral operators is increased, despite the ill-conditioned form of a system of the type (2.6).

Remark. The above approach for reducing boundary-value problem (1.1)–(1.2) to a system of boundary integral equations of the first kind (1.7) can easily be transferred to the case when the attenuation in the electroelastic medium is taken into account using the concept of complex moduli, by making the replacement

$$c_{mjkl} \to c_{mjkl}(i\omega), \quad e_{mjk} \to e_{mjk}(i\omega), \quad э_{mj} \to э_{mj}(i\omega)$$

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